11.10 TAYLOR SERIES

$$f(x) \approx T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

IF $a=0$
WE CALL THE SERIES
A McLAURIN SERIES

TAYLOR POLYNOMIAL OF DEGREE k

$$T_k(x) = \underbrace{\sum_{n=0}^{k} \frac{f^{(n)}(a)}{n!}(x-a)^n}_{k+1 \text{ TERMS}}$$

a

eg. FIND TAYLOR SERIES OF $f(x)=e^x$ ABOUT $x=0$

n	$f^{(n)}(x)$	$f^{(n)}(a)/n! * (x-a)^n$
0	e^x	$e^0 = 1/0! * (x-0)^0 = \frac{1}{1}x^0 = 1$
1	e^x	$e^0 = 1/1! * (x-0)^1 = \frac{1}{1}x^1 = x$
2	e^x	$1/2! * (x-0)^2 = \frac{1}{2!}x^2$
3	e^x	$1/3! * (x-0)^3 = \frac{1}{3!}x^3$
4	e^x	$1/4! * (x-0)^4 = \frac{1}{4!}x^4$
5	e^x	$1/5! * (x-0)^5 = \frac{1}{5!}x^5$
n	e^x	$1/n! * (x-0)^n = \frac{1}{n!}x^n$

$$T(x) = \sum_{n=0}^{\infty} \frac{1}{n!}x^n = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots$$

INTERVAL OF CONVERGENCE FOR $T(x)$ FOR $e^x = (-\infty, \infty)$

$$T(x) = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \lim_{n\to\infty} \left| \frac{x}{n+1} \right| = 0 < 1$$

FOR ALL $x \in \mathbb{R}$

FIND TAYLOR SERIES OF $f(x) = \ln x$ ABOUT ~~$x=0$~~

← $f(0)$ DNE

$x = 1$

a

TRY $x=e$ YOURSELF

n	$f^{(n)}(x)$	$f^{(n)}(1)/n! * (x-1)^n$
0	$\ln x$	0
1	x^{-1}	$1/1! * (x-1)^1$
2	$(-1)x^{-2}$	$-1/2! * (x-1)^2$
3	$(-2)(-1)x^{-3}$	$(-2)(-1)/3! * (x-1)^3$
4	$(-3)(-2)(-1)x^{-4}$	$(-3)(-2)(-1)/4! * (x-1)^4$
n		$(-1)^{n-1}(n-1)!/n! * (x-1)^n = \frac{(-1)^{n-1}}{n}(x-1)^n$

n STARTS AT 1 → 1

$$T(x) = \sum_{n=\cancel{0}\,1}^{\infty} \frac{(-1)^{n-1}}{n}(x-1)^n = \sum_{n=\cancel{0}\,1}^{\infty} \frac{(-1)^{n+1}}{n}(x-1)^n \approx (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \ldots$$

INTERVAL OF CONVERGENCE OF $T(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(x-1)^n$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+2}(x-1)^{n+1}}{n+1}\cdot\frac{n}{(-1)^{n+1}(x-1)^n}\right| = \lim_{n\to\infty}\left|\frac{1}{1+\frac{1}{n}}(x-1)\right|$$

$$= |x-1|\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}$$

$$= |x-1|\cdot 1 = |x-1| < 1$$

$$-1 < x-1 < 1$$

$$0 < x < 2$$

$x=0$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(0-1)^n$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^{2n+1}}{n}$$

$$= \sum_{n=1}^{\infty}-\frac{1}{n}$$

$$= -\sum_{n=1}^{\infty}\frac{1}{n}$$ DIV (p-SERIES)

$p=1\leq 1$

$x=2$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(2-1)^n$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$$ CONV (ALT)

$\frac{1}{n}>0$ ON $[1,\infty)$

$\lim_{n\to\infty}\frac{1}{n}=0$

$f'(x)=-\frac{1}{x^2}<0$ ON $[1,\infty)$

$\{\frac{1}{n}\}$ DECR

INTERVAL OF CONVERGENCE $=(0,2]$

FIND THE FIRST 3 NON-ZERO TERMS TAYLOR SERIES FOR $\tan x$ ABOUT $x = 0$

n	$f^{(n)}(x)$	$f^{n}(0)/n! * x^{n}$
0	$\tan x$	0
1	$\sec^2 x$	$1/1! * x^1$
2	$2\sec^2 x \tan x$	0
3		
4		0
5		